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Shaft Center Orbit
for
Dynamically Loaded
Journal Bearings

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ABSTRACT

The aim of this work is to demonstrate how to utilize the bearings damping coefficients to estimate the orbit for a dynamically loaded journal bearing. The classical method for this analysis was developed by Booker in 1965 [1]and described further in 1972 [2]. Several authors have refined this method over the years. In 1966 Jorgen W. Lund [5]published an approach to find the dynamic coefficients of a journal bearing by a first order perturbation of the Reynold's equation. These coefficients made it possible to perform a rotor-bearing stability analysis for a statically loaded bearing. In the mid seventies Jorgen W. Lund pointed out in lecture notes that the dynamic damping coefficients of the bearing could be used to find the shaft orbit for dynamically loaded bearings. For simplicity the "Short-Width-Journal-Bearing Theory" is used as a basis for finding the damping coefficients in this work, but the method is general and the damping coefficients could have been found also by numerical solutions.

Keywords: journal bearing, dynamic load, orbit.

1 RECIPROCATING MACHINERY

Hydrodynamic bearings for steam turbines, radial compressors e.t.c. are primarily designed to carry a static load. The dynamic forces acting are usually not important in this type of machinery.

In combustion engines and piston compressors the bearings are subject to considerable dynamic loads. These loads stem from the combustion or compression process and from the inertia forces of pistons and connecting rods.

Due to the periodicity of the dynamic load the journal center describes a closed loop. The loop geometry is depending on bearing geometry and operating conditions. An important design issue is to ensure that there is no contact between journal and sleeve under operation. Several papers have been published in this field over the years. See for example [6] [7] [3] for an overview.

2 REYNOLDS EQUATION FOR DYNAMIC LOAD

The dynamic forces in reciprocating machinery are time dependent and periodic. If a stationary X-Y-coordinate system is introduced the external load W_r acting on the bearing at time t can be given as the the components W_x and W_y see figure 1. Both components are functions of time and periodic with a period length depending on the actual machinery.

In reciprocating machinery both journal and sleeve may rotate and the angular speeds are designated ω_a and ω_b respectively. These speeds may also be time dependent as it is the case for the crank-connecting rod bearing, but again the variation will be periodic. If it is assumed that the lubricant is incompressible Reynolds equation takes the form:

$$\frac{\partial}{R\partial\theta}\left(\frac{h^3}{12\eta}\frac{\partial p}{R\partial\theta}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial z}\right)=\frac{1}{2}(\omega_a+\omega_b)\frac{\partial h}{\partial\theta}+\frac{\partial h}{\partial t}=\frac{1}{2}\omega\frac{\partial h}{\partial\theta}+\frac{\partial h}{\partial t} \tag{1}$$

The load carrying capacity (the hydro-dynamic forces) F_x and F_y are found from an integration of the film pressure, and they are as such functions of the instant journal center position measured from the sleeve center, the squeeze speed and the effective angular velocity $\omega = \omega_a + \omega_b$:

$$F_x = F_x(x, y, \dot{x}, \dot{y}, \omega) \tag{2}$$

$$F_y = F_x(x, y, \dot{x}, \dot{y}, \omega) \tag{3}$$

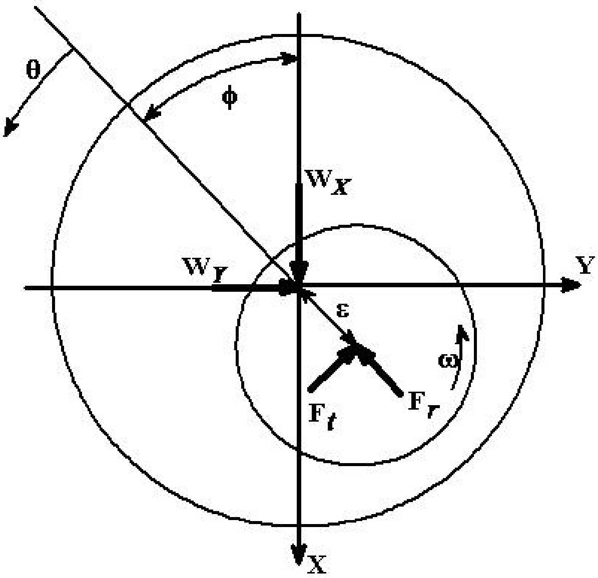

Figure 1: Coordinate system and forces.

If M is the effective mass related to the journal we can write the equations for the motion as:

$$M\frac{\partial^2 x}{\partial^2 t} = W_x - F_x \tag{4}$$

$$M\frac{\partial^2 y}{\partial^2 t} = W_y - F_y \tag{5}$$

The external loads W_x and W_y includes the inertia forces from the motion of the mechanical components. The inertia forces coming from the relative motion between the journal and the sleeve are, compared to this, relatively small. The equations therefore simplifies to:

$$W_x = F_x \tag{6}$$

$$W_y = F_y \tag{7}$$

F_x and F_y are strongly nonlinear functions of their arguments and these equations can not be solved directly. The calculation must be performed iteratively. The period T of the external load is divided into small time intervals Δt. For each time step the equations are solved so that force equilibrium is obtained.

3 THE ITERATIVE PROCEDURE

Let k be the index so that at time $t = t_k$ the coordinates of the journal are x_k and y_k and the external forces are W_{x_k} and W_{y_k}, and the effective angular velocity is ω_k. The unknowns are then $\dot{x}_k$ and $\dot{y}_k$ and they must be estimated so that the equations are fulfilled. This is done by iteration. In the j'th velocity iteration the calculated values of the velocities are $\dot{x}_{k_j}$ and $\dot{y}_{k_j}$. These values are increased with $\Delta\dot{x}_j$ and $\Delta\dot{y}_j$, whereby the hydrodynamic forces may be expressed by a Taylor expansion:

$$F_x = F_x(x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k) + B_{xx_j}\Delta\dot{x}_j + B_{xy_j}\Delta\dot{y}_j \tag{8}$$

$$F_y = F_x(x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k) + B_{yx_j}\Delta\dot{x}_j + B_{yy_j}\Delta\dot{y}_j \tag{9}$$

where B_{ij} may be derived as:

$$B_{xx_j} = (\frac{\partial F_x}{\partial \dot{x}})_{x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k} \tag{10}$$

$$B_{yx_j} = (\frac{\partial F_y}{\partial \dot{x}})_{x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k} \tag{11}$$

$$B_{xy_j} = (\frac{\partial F_x}{\partial \dot{y}})_{x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k} \tag{12}$$

$$B_{yy_j} = (\frac{\partial F_y}{\partial \dot{y}})_{x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k} \tag{13}$$

These four damping coefficients may be found by a perturbation of Reynolds equation as described in [5], or if the short-width-journal bearing theory is used the coefficients may be found by direct differentiation.

$$\begin{pmatrix} B_{xx_j} & B_{xy_j} \\ B_{yx_j} & B_{yy_j} \end{pmatrix} \begin{pmatrix} \Delta\dot{x}_j \\ \Delta\dot{y}_j \end{pmatrix} = \begin{pmatrix} W_{x_k} - F_x(x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k) \\ W_{y_k} - F_y(x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k) \end{pmatrix} \tag{14}$$

The equations are solved to find $\Delta\dot{x}_j$ and $\Delta\dot{y}_j$ and in the next iteration we get:

$$\dot{x}_{k_{j+1}} = \dot{x}_{k_j} + \Delta\dot{x}_j \tag{15}$$

$$\dot{y}_{k_{j+1}} = \dot{y}_{k_j} + \Delta\dot{y}_j \tag{16}$$

As a convergence criteria one could use:

$$\frac{|W_{x_k} - F_x(x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k| + |W_{y_k} - F_y(x_k, y_k, \dot{x}_{k_j}, \dot{y}_{k_j}, \omega_k|}{|W_{x_k}| + |W_{y_k}|} < 10^{-5} \tag{17}$$

When convergence in the iterations has been achieved as defined in the equations the calculations proceed to the next time step where the time is now $t = t_{k+1} = t_k + \Delta t$. The journal center coordinates are found from:

$$x_{k+1} = x_k + \Delta t \dot{x}_k \tag{18}$$

$$y_{k+1} = y_k + \Delta t \dot{y}_k \tag{19}$$

Now the calculation process proceeds until the orbit of the journal is repeated in consecutive time periods. It may take several time periods of the dynamic load until the orbit is stable, [3]. For a uniform clearance of the bearing usually less than two strokes are needed to obtain the cyclic orbit. The number of iteration cycles are however strongly depending of the circumferential clearance geometry of the bearing.

4 THE SHORT-WIDTH-JOURNAL-BEARING THEORY

The present analysis is based on the short-width-journal-bearing theory primarily because it is simple, but also because it is superior to the numerical solutions when the eccentricity ratio becomes very high. Introducing the following dimension less variables:

$$\theta = \frac{x}{R} \tag{20}$$

$$\zeta = \frac{z}{R} \tag{21}$$

$$\overline{p} = \frac{p}{6\eta\omega(\frac{R}{C})^2} \tag{22}$$

$$\overline{h} = \frac{h}{C} \tag{23}$$

$$\tau = \nu t \tag{24}$$

Where $\omega = \omega_a + \omega_b$. Assuming that the short-width-journal-bearing theory applies we can write Reynolds equation as:

$$\frac{\partial}{\partial\zeta}\left(\overline{h}^3 \frac{\partial\overline{p}}{\partial\zeta}\right) = \frac{\partial\overline{h}}{\partial\theta} + 2\frac{\nu}{\omega}\frac{\partial\overline{h}}{\partial\tau} \tag{25}$$

The forces acting on the journal from the fluid film is found by integration:

$$f_r = \frac{F_r}{\eta NDL(\frac{R}{C})^2} = \frac{6\pi}{\frac{L}{D}} \int_0^{\frac{L}{D}} \int_{\theta_1}^{\theta_2} -p\cos\theta d\theta d\zeta \tag{26}$$

$$f_t = \frac{F_t}{\eta NDL(\frac{R}{C})^2} = \frac{6\pi}{\frac{L}{D}} \int_0^{\frac{L}{D}} \int_{\theta_1}^{\theta_2} p\sin\theta d\theta d\zeta \tag{27}$$

The total load carrying capacity of the bearing is:

$$W = \sqrt{{F_r}^2 + {F_t}^2} \tag{28}$$

For later reference the Sommerfeld number is defined:

$$S(t) = \frac{\eta N(t)DL}{W(t)} \left(\frac{R}{C}\right)^2 \tag{29}$$

The dimension less film thickness can be expressed as:

$$\overline{h}(\theta) = 1 + \varepsilon\cos\theta = 1 + \varepsilon\cos(\theta' - \phi) \tag{30}$$

$$\frac{\partial\overline{h}}{\partial\theta} = -\varepsilon\sin\theta \tag{31}$$

where $\theta = \theta' - \phi$. For the squeeze term we find that:

$$\frac{\nu}{\omega}\frac{\partial\overline{h}}{\partial\tau} = \frac{\nu}{\omega}[\frac{d\varepsilon}{d\tau}\cos\theta + \varepsilon\frac{d\phi}{d\tau}\sin\theta] = \dot{\varepsilon}\cos\theta + \varepsilon\dot{\phi}\sin\theta \tag{32}$$

where:

$$\dot{\varepsilon} = \frac{\nu}{\omega}\frac{d\varepsilon}{d\tau} = \frac{\nu}{\omega}\frac{d\varepsilon}{\nu dt} = \frac{d\varepsilon}{\omega dt} \tag{33}$$

$$\dot{\phi} = \frac{\nu}{\omega}\frac{d\phi}{d\tau} = \frac{\nu}{\omega}\frac{d\phi}{\nu dt} = \frac{d\phi}{\omega dt} \tag{34}$$

Integrating Reynolds equation twice and using the boundary conditions $\overline{p} = 0$ for $\zeta = \pm\frac{L}{D}$ and $\frac{\partial\overline{p}}{\partial\zeta} = 0$ for $\zeta = 0$ gives:

$$\overline{p} = \frac{1}{2}\left(\left(\frac{L}{D}\right)^2 - \zeta^2\right)\frac{1}{\overline{h}^3}\left(\varepsilon\sin\theta - 2\dot{\varepsilon}\cos\theta - 2\varepsilon\dot{\phi}\sin\theta\right) \tag{35}$$

The load carrying capacity can be found according to eq. 26 and eq. 27.

$$f_\varepsilon = \pi \left(\frac{L}{D}\right)^2 \left[\varepsilon\left(1-2\dot{\phi}\right)\left(-2\int_{\theta_1}^{\theta_2}\frac{\cos\theta\sin\theta}{\overline{h}^3}d\theta\right) + 2\dot{\varepsilon}\left(2\int_{\theta_1}^{\theta_2}\frac{\cos^2\theta}{\overline{h}^3}d\theta\right)\right] \quad (36)$$

$$f_\phi = \pi \left(\frac{L}{D}\right)^2 \left[\varepsilon\left(1-2\dot{\phi}\right)\left(2\int_{\theta_1}^{\theta_2}\frac{\sin^2\theta}{\overline{h}^3}d\theta\right) + 2\dot{\varepsilon}\left(-2\int_{\theta_1}^{\theta_2}\frac{\cos\theta\sin\theta}{\overline{h}^3}d\theta\right)\right] \quad (37)$$

f_ε is acting along the line connecting sleeve and journal center and f_ϕ is perpendicular to it. The Sommerfeld substitution may be used for an analytical solution of the integrals listed above.

From eq. 35 it can bee seen that $\overline{p}$ is positive when

$$\varepsilon\left(1-2\dot{\phi}\right)\sin\theta - 2\dot{\varepsilon}\cos\theta \geq 0 \quad (38)$$

The film with positive pressure starts at $\theta = \theta_1$ and ends at $\theta = \theta_2 = \theta_1 - \pi$:

$$\theta_1 = \arctan\left(\frac{2\dot{\varepsilon}}{\varepsilon\left(1-2\dot{\phi}\right)}\right) \quad (39)$$

Generally the the force components are depending on the following variables:

$$f_\varepsilon = f_\varepsilon\left(\varepsilon, \phi, \dot{\varepsilon}, \dot{\phi}\right) \quad (40)$$

$$f_\phi = f_\phi\left(\varepsilon, \phi, \dot{\varepsilon}, \dot{\phi}\right) \quad (41)$$

For small amplitudes from the current journal position we can make a first order Taylor expansion:

$$f_\varepsilon = f_{\varepsilon 0} + \left(\frac{\partial f_\varepsilon}{\partial\varepsilon}\right)_0 \triangle\varepsilon + \left(\frac{\partial f_\varepsilon}{\varepsilon_0\partial\phi}\right)_0 \varepsilon_0\triangle\phi + \left(\frac{\partial f_\varepsilon}{\partial\dot{\varepsilon}}\right)_0 \triangle\dot{\varepsilon} + \left(\frac{\partial f_\varepsilon}{\varepsilon_0\partial\dot{\phi}}\right)_0 \varepsilon_0\triangle\dot{\phi} \quad (42)$$

$$f_\phi = f_{\phi 0} + \left(\frac{\partial f_\phi}{\partial\varepsilon}\right)_0 \triangle\varepsilon + \left(\frac{\partial f_\phi}{\varepsilon_0\partial\phi}\right)_0 \varepsilon_0\triangle\phi + \left(\frac{\partial f_\phi}{\partial\dot{\varepsilon}}\right)_0 \triangle\dot{\varepsilon} + \left(\frac{\partial f_\phi}{\varepsilon_0\partial\dot{\phi}}\right)_0 \varepsilon_0\triangle\dot{\phi} \quad (43)$$

The damping coefficients may be written as:

$$\overline{B}_{rr} = \left(\frac{\partial f_\varepsilon}{\partial\dot{\varepsilon}}\right)_{\varepsilon=\varepsilon_0} \quad (44)$$

$$\overline{B}_{rt} = \left(\frac{\partial f_\varepsilon}{\varepsilon_0 \partial \dot{\phi}}\right)_{\varepsilon=\varepsilon_0} \tag{45}$$

$$\overline{B}_{tr} = -\left(\frac{\partial f_\phi}{\partial \dot{\varepsilon}}\right)_{\varepsilon=\varepsilon_0} \tag{46}$$

$$\overline{B}_{tt} = -\left(\frac{\partial f_\phi}{\varepsilon_0 \partial \dot{\phi}}\right)_{\varepsilon=\varepsilon_0} \tag{47}$$

Giving:

$$\begin{bmatrix} f_\varepsilon - f_{\varepsilon 0} \\ f_\phi - f_{\phi 0} \end{bmatrix} = \begin{bmatrix} \overline{B}_{rr} & \overline{B}_{rt} \\ \overline{B}_{tr} & \overline{B}_{tt} \end{bmatrix} \begin{bmatrix} \triangle \dot{\varepsilon} \\ \varepsilon_0 \triangle \dot{\phi} \end{bmatrix} \tag{48}$$

The load carrying capacity and the damping coefficients are expressed in a co-ordinate system which moves with the journal. This is not convenient for the current purpose. A transformation to the stationary co-ordinate system with the x-axis pointing vertically downwards is performed:

$$f_x = f_\varepsilon \cos\phi_0 + f_\phi \sin\phi_0 \tag{49}$$

$$f_y = f_\varepsilon \sin\phi_0 - f_\phi \cos\phi_0 \tag{50}$$

The amplitudes from the current equilibrium position becomes:

$$\begin{bmatrix} \triangle \overline{x} \\ \triangle \overline{y} \end{bmatrix} = \begin{bmatrix} \overline{x} - \overline{x}_0 \\ \overline{y} - \overline{y}_0 \end{bmatrix} = \begin{bmatrix} \cos\phi_0 & -\sin\phi_0 \\ \sin\phi_0 & \cos\phi_0 \end{bmatrix} \begin{bmatrix} \triangle \varepsilon \\ \varepsilon_0 \triangle \phi \end{bmatrix} \tag{51}$$

and further:

$$\begin{bmatrix} \triangle \varepsilon \\ \varepsilon_0 \triangle \phi \end{bmatrix} = \begin{bmatrix} \cos\phi_0 & \sin\phi_0 \\ -\sin\phi_0 & \cos\phi_0 \end{bmatrix} \begin{bmatrix} \triangle \overline{x} \\ \triangle \overline{y} \end{bmatrix} \tag{52}$$

The forces in the stationary coordinate system become:

$$\begin{bmatrix} f_x - f_{x0} \\ f_y - f_{y0} \end{bmatrix} = \begin{bmatrix} \overline{B}_{xx} & \overline{B}_{xy} \\ \overline{B}_{yx} & \overline{B}_{xy} \end{bmatrix} \begin{bmatrix} \triangle \dot{\overline{x}} \\ \triangle \dot{\overline{y}} \end{bmatrix} \tag{53}$$

Applying the transformations gives:

$$\overline{B}_{xx} = \overline{B}_{rr} \cos^2\phi_0 + \overline{B}_{tt} \sin^2\phi_0 - \left(\overline{B}_{rt} + \overline{B}_{tr}\right) \cos\phi_0 \sin\phi_0 \tag{54}$$

$$\overline{B}_{xy} = \overline{B}_{rt} \cos^2\phi_0 - \overline{B}_{tr} \sin^2\phi_0 + \left(\overline{B}_{rr} - \overline{B}_{tt}\right) \cos\phi_0 \sin\phi_0 \tag{55}$$

$$\overline{B}_{yx} = \overline{B}_{tr} \cos^2\phi_0 - \overline{B}_{rt} \sin^2\phi_0 + \left(\overline{B}_{rr} - \overline{B}_{tt}\right) \cos\phi_0 \sin\phi_0 \tag{56}$$

$$\overline{B}_{yy} = \overline{B}_{tt} \cos^2\phi_0 + \overline{B}_{rr} \sin^2\phi_0 + \left(\overline{B}_{rt} + \overline{B}_{tr}\right) \cos\phi_0 \sin\phi_0 \tag{57}$$

5 RESULTS

For presentation purposes a standard bearing was chosen for analysis. The connecting rod bearing of the Hornsby 6VEV-X Mk III 4 stroke diesel engine was adapted. This bearing has been widely used for analysis and extensively investigated by experiments [4]. The input values are shown in table 5.

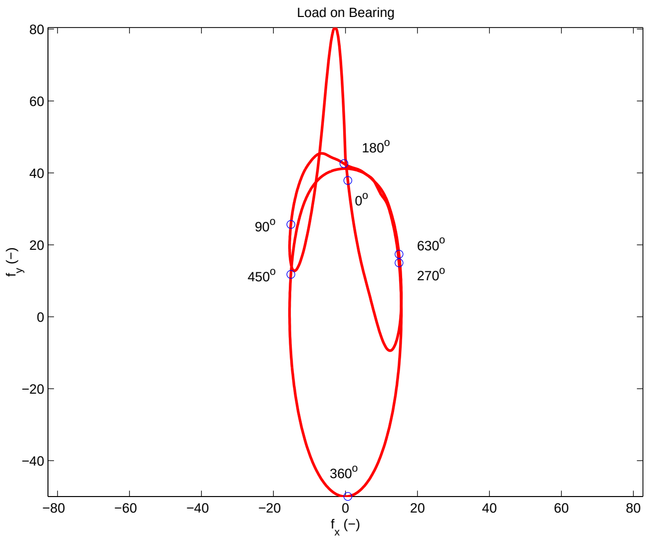

Figure 2: *Load on Bearing (connecting rod coordinate system)*

Figure 2 shows the loading diagram for the calculation performed.

Speed (N)	10 cps.
Clearance (C)	82.6 μm
Diameter (D)	0.203 m
$\frac{1}{2}$ Length (L)	0.0575m
Viscosity (η)	0.015Pas

Table 1: Input values for the calculation

Additional data required for this analysis may be found in [4]. Figure 3 shows the corresponding results of the calculations. The results are in good agreement with the results obtained by Booker [4]

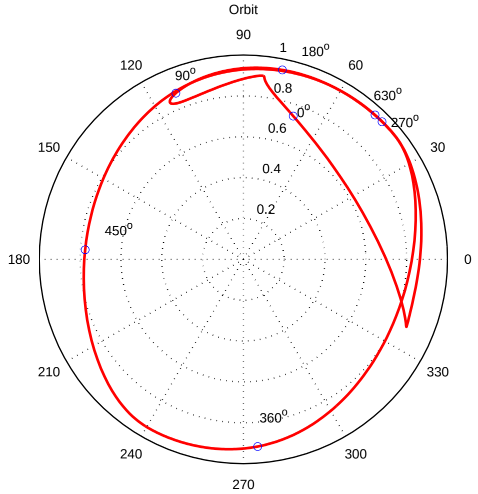

Figure 3: *Journal orbit (connecting rod coordinate system)*

6 CLOSURE

The described calculation method is general and can be applied for any journal bearing geometry.

The method is robust and has a high convergence rate. The results obtained are very similar to the results obtained with other more complicated methods. Another advantage using the short-width-bearing theory is that the solution is obtained in a fraction of the time it takes for a numerical approach. Therefore this method may be advantageous in an analysis of new design proposals of a journal bearing.

Appendix A. Nomenclature

B Damping coefficient (Ns/m)
$\overline{B}$ Dim. less damping coefficient $(-)$
C Radial clearance (m)
D Bearing diameter (m)
F Fluid film force on journal (N)
f Dim. less fluid film force on journal $(-)$
h Film thickness (m)
$\overline{h}$ Dim. less film thickness $(-)$
L Bearing width (m)
N Rotational speed (cps)
p Fluid film pressure (Nm^{-2})
$\overline{p}$ Dim. less fluid film pressure $(-)$
R Bearing radius (m)
W Applied external load (N)
ε Eccentricity ratio $(-)$
θ Circumferential co-ordinate (rad)
ϕ Attitude angle (rad)
ω_a Angular speed of journal $(rads^{-1})$
ω_b Angular speed of sleeve $(rads^{-1})$
ν Reference angular speed $(1/s)$

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